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AN EVALUATION OF BULEEV'S MODEL OF TURBULENT EXCHANGE

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Abstract—An evaluation is made of a "universal" model of turbulent exchange due to Buleev [1] which gives expressions for all the components of the turbulent stress tensor and of the heat- or mass-flux vector. Recent successful applications of this model in a variety of cases have made it worthy of much attention. Here the predicted eddy diffusivities of momentum and heat or mass for flow in several configurations are compared with experiment. The evaluation and comparison do not substantiate the claim of universality. In particular, the predicted tangential eddy diffusivities of heat or mass in a plain tube are very different from the experimental values and an inherent restriction in the model prevents any improvement.

NOMENCLATURE

b_1, b_2, b_3,	constants in turbulent exchange process of mole;
c,	coefficient;
c_1, c_2,	constants;
c_p,	specific heat at constant pressure;
D,	domain of integration;
d,	diameter of mole;
f_0, f_1, f_2,	functions of mole behaviour;
k,	thermal conductivity;
L, L',	mixing lengths;
m,	coefficient;
n,	direction of mean velocity;
p_1, p_2,	argument;
Pr,	Prandtl number;
q,	argument;
r,	radius of tube;
R,	radius of mole;
Re,	Reynolds number;
s,	radius of spherical co-ordinate system, centred on M_0;
T',	temperature fluctuation;
$\bar{T}$,	mean temperature;
$\bar{u}, \bar{v}, \bar{w}$,	mean velocity components in x, y, z directions;
u', v', w',	fluctuating velocity components in x, y, z directions;
u^+,	$u/\sqrt{(\tau_\infty/\rho)}$;
V,	mean velocity;
V',	fluctuating velocity;
V^*,	instantaneous velocity equal to $V' + V$;
x, y, z,	rectangular co-ordinates;
y^+,	$y\sqrt{(\tau_\infty/\rho)}/\nu$.

Greek symbols

α,	dimensional coefficient;
β,	dimensional coefficient;
λ,	mean free path of mole;
μ,	dimensionless coefficient;
δ,	dimensionless coefficient;
ξ,	dimensionless co-ordinate $(y - y_o)/\alpha L_o$;
ζ,	dimensionless co-ordinate $(z - z_o)/\alpha L_o$;
ε,	eddy diffusivity;
ρ,	density;
ν,	kinematic viscosity;
τ,	shear stress.

Subscripts

b,	bulk or average value;
i,	inner radius or wall value;
o,	outer radius of wall value or value at M_o;
h,	heat;
m,	mass;
ω,	tangential.

INTRODUCTION

MANY attempts have been made to set up models of turbulent exchange based on an analogy between the eddy of turbulent flow and the molecule of the kinetic theory of gases. Few of these go further than a prediction of cross-stream momentum or heat transfer. The Buleev [1] analysis, however, gives expressions from which each one of the terms of the turbulent stress tensor and heat- or mass-flux vector may be calculated and it has been applied by him to several complex cases [2, 3]. There appears to be no restriction on the generality of these results and they are applicable, at least in principle, to ducts of arbitrary shape, and boundary layers and to recirculating flows and separation. If it were true that all the Reynolds stresses and all the fluxes could be calculated in any turbulent flow as Buleev describes, then the modelling of turbulence would no longer be a problem. Support for the Buleev model has been given recently by Ramm and Johannsen who used it to calculate flow and heat transfer in tubes, parallel-plate channels, annuli [4], and in the space between rod bundles [5]. Also their calculations [6] of the tangential eddy diffusivity of heat or mass in a plain tube showed excellent agreement with experiment. Most recently [7, 8], they again applied the model to liquid metal heat transfer, apparently most successfully.

Considering that many decades of research on the modelling of turbulence has produced results only of rather limited applicability, these verifications make Buleev's model worthy of much attention. Indeed Reynolds [9] in his important review of turbulence modelling considered it to be the most advanced of those based on the mixing-length hypothesis.

This work consists of an evaluation of the theory and of comparisons between its prediction and experiment in four simple cases.

DESCRIPTION AND EVALUATION OF BULEEV'S MODEL

Buleev's results for the components of the stress tensor $\overline{u'^2}$, $\overline{u'v'}$ etc., and for the components of the heat-flux vector $\overline{u't'}$, $\overline{v't'}$, $\overline{w't'}$ are given in the Appendix, equations (A1–A8). Each Reynolds stress or flux term is calculated from a complicated relationship involving a length scale, the gradient of the mean flow, the gradient of each of the components of the mean flow and constants and functions which are derived from considering the momentum- and heat-exchange integrals taken over the volume surrounding the point of interest. The heat-flux terms involve the local temperature gradient. These results are obtained as follows.

Buleev's model pictures a "mole" or eddy emanating from some arbitrary point, M, contributing to the velocity and temperature fluctuations at the point of interest, M_o. The instantaneous values at M_o are as given by equations (A9). Two criticisms may be made. Each of the functions f_0, f_1 and f_2, which are determined from the exchange processes of the moving eddy, is presumed to have a value *less* than unity. This cannot be generally true. Also it is inconsistent to relate the temperature fluctuations to the mean temperature difference only. Appropriate multiplication gives equations (A10), and to calculate the stress tensor and heat-flux vector at M_o the functions F_1 and F_2 are integrated over the spatial domain D surrounding M_o. That is

$$\rho\overline{u'w'} = \int_D F_1(MM_o)\phi(M)\,\mathrm{d}\tau \tag{1}$$

$$\rho c_p\overline{u'T'} = \int_D F_2(MM_o)\phi(M)\,\mathrm{d}\tau \tag{2}$$

where $\phi(M)$ is a weighting function.

This is most challengeable, since the time-average values are being equated to an average over a region of space. The process has a tenuous kinship with integrations performed in applying the kinetic theory of gases. But there, the functions are probability distributions of position, speed and direction; whereas here, the integration is applied to a deterministic relationship between values at two identified points.

The functions f_0, f_1 and f_2 are deduced as follows. The instantaneous changes in the velocity components and temperature of a mole proceeding from M to M_o are given by equations (A11) and it is clear that they are equivalent to an assumption that the velocity and temperature fluctuations decay exponentially with time. The coefficients A_1 and A_2 are given by

$$A_1 = b_1\frac{\nu}{R} + b_3\frac{\nu}{R} \tag{3}$$

and

$$A_2 = b_2\frac{K}{R} + b_3\delta\frac{\nu}{R}. \tag{4}$$

Here, b_1, b_2, b_3 and δ are dimensionless coefficients and the equations for diameter, mean free path and fluctuation in speed of the mole, equations (A12), introduce three more, so that at this stage there are *seven* undetermined coefficients. The first terms of A_1 and A_2 were regarded as representing a molecular mechanism of exchange, whilst the second represented a convection of smaller moles breaking off the mole. It was assumed that b_1 and b_2 depend on the Reynolds number of the mole, $2RV'/\nu$; b_2 depends on this and on the Prandtl number also. Three hardly plausible assumptions are involved. Firstly, that the time element $\mathrm{d}t$ can be replaced by $\mathrm{d}r/V'$ where V' is the fluctuating component only; secondly, that V' is in any case constant between M and M_o which is contrary to the basic model and, thirdly, that $\bar{u}$, $\bar{w}$ vary linearly between M and M_o, which cannot be true in general. On solving the equations with these assumptions f_0, f_1 and f_2 are as given by equation (A13); but in deducing f_2 Buleev neglected a term similar to f_0.

The weighting function $\phi(M)$ equation (A14) is deduced from a different analogy namely that of neutron-flux attenuation. Buleev used the second analogy as though the moles were emanating from M_o and by equating k_o, the mean free path in the neutron flux attenuation analogy with λ, the mean free path in the kinetic theory analogy, it can be expressed as a function of the length scale, L. This introduces the eighth unknown coefficient, c.

Appropriate substitution for f_0, f_1, f_2 and ϕ gives expressions for the Reynolds stresses and heat flux components. The results however do not agree with Buleev's final forms as given in the appendix. A further simplification is made that the products of cosines of different arguments can be ignored and this entirely destroys confidence in the reality of the model, even if we accept individual steps uncritically. Products of cosines of different arguments arise in the analysis from products of the fluctuating components: products of cosines of the same argument arise from products involving fluctuating components and mean values. In the basic derivation of the Reynolds stresses the time averaging retains the former whilst the latter are identically zero. This simplification has dropped the terms which contribute to the Reynolds stresses whilst retaining those which do not. Clearly any justification for the Buleev model can only be heuristic.

APPLICATION OF BULEEV'S MODEL TO SIMPLE CASES

For a fully developed flow Buleev's equations give $-\rho\overline{u'w'}$ and $-\rho\overline{w'v'}$ as identically zero but

$$-\rho\overline{u'v'} = \rho\mu\int L\left[\frac{\partial V}{\partial n}\right] f_0 f_1 s\phi\cos^2(s,y)\left[\frac{\partial u}{\partial y}\right]\mathrm{d}\tau. \tag{5}$$

With the assumption that the mean velocity gradient everywhere in the domain of integration is equal to the value at M_o this can be written as an eddy diffusivity. Thus,

$$\varepsilon_{m,y} = \rho\mu \int L\left[\frac{\partial V}{\partial n}\right] f_0 f_1 \phi s \cos^2(s, y)\,d\tau \qquad (6)$$

and by analogy, though without other justification,

$$\varepsilon_{m,z} = \rho\mu \int L\left[\frac{\partial V}{\partial n}\right] f_0 f_1 \phi s \cos^2(s, z)\,d\tau. \qquad (7)$$

These two equations are the starting point for the calculation of eddy diffusivities. Two simplifying assumptions are made. Firstly, that all functions and values in the three dimensional integral are assumed constant on a plane perpendicular to the y axis, which can be true only in a very special case. Secondly, the resulting two-dimensional integral over the plane y–z can be replaced by an integral taken along a particular line in this plane which makes an angle with the y axis of between 34 and 36°. The secant of this angle is introduced as an empirical coefficient, m. Introducing the coordinates ξ and ζ, then,

$$\varepsilon_{m,y} = c\mu\alpha L_o^2 \int_{-\infty}^{\infty} \frac{L}{L_o}\left[\frac{\partial \bar{u}}{\partial n}\right] f_0 f_1 G(\xi)\,d\xi \qquad (8)$$

and

$$\varepsilon_{m,z} = c\mu\alpha L_o^2 \int_{-\infty}^{\infty} \frac{L}{L_o}\left[\frac{\partial \bar{u}}{\partial n}\right] f_0 f_1 G(\zeta)\,d\zeta \qquad (9)$$

where another simplification has been made in that values in the integral are taken along the y, z axes and not along the line of constant angle in the plane. The integration over a volume of the flow has been reduced to a line integral, only. The function G includes the attenuation function ϕ.

When considering the terms of the heat flux vector it is soon realised that for a fully developed flow the expressions for $-\rho c_p\overline{u'T'}$, $-\rho c_p\overline{v'T'}$ and $-\rho c_p\overline{w'T'}$ depend on all the components of the temperature gradients and vary accordingly. Also $-\rho c_p\overline{u'T'}$ involves $\partial\bar{u}/\partial y$ and it is easy to imagine cases in which the effect would lead to anomalous results.

The eddy diffusivity of heat expressions become

$$\varepsilon_{h,y} = c\mu\alpha L_o^2 \int_{-\infty}^{\infty} \frac{L}{L_o}\left[\frac{\partial \bar{u}}{\partial n}\right] f_0 f_2 G(\xi)\,d\xi \qquad (10)$$

and

$$\varepsilon_{h,z} = c\mu\alpha L_o^2 \int_{-\infty}^{\infty} \frac{L}{L_o}\left[\frac{\partial \bar{u}}{\partial n}\right] f_0 f_2 G(\zeta)\,d\zeta \qquad (11)$$

when all the appropriate simplifications and substitutions have been made.

In evaluating the eddy diffusivities we have to give values to the coefficients α, β, μ which arise from the kinetic theory model; to b_1, b_2, b_3 and δ which arise from the supposed exchange process between the mole and its surroundings and to c (or m) which arises from the simplification of the volume integral. The coefficient δ is given the value unity which is in essence an assumption that Reynolds' analogy applies to the micro-particles which break off the mole, whilst m is taken as 1.25. Buleev proposed $\alpha = 0.33$, $\mu\alpha = 0.72$, $b_1 = 0.9$, $b_2 = b_1 \times Pr^{-0.33}$ and $b_3 = 3.5$. In arriving at these values it was assumed that the mean free path and diameter of the mole are equal, that is, $\beta = \alpha$.

In terms of these coefficients, the arguments of $f_0(q_1\xi)$, $f_1(q_1\xi)$ and $f_2(q_2\xi)$ are

$$q_1 = \left[\frac{12}{\beta^2\mu}(b_1+b_3)m\alpha\right]\frac{\nu L_o}{L^3\left[\frac{\partial V}{\partial n}\right]} \qquad (12)$$

$$q_2 = \left[\frac{12}{\beta^2\mu}\left(\frac{b_1}{Pr^{n-1}}+b_3\right)m\alpha\right]\frac{\nu L_o}{L^3\left[\frac{\partial V}{\partial n}\right]} \qquad (13)$$

and it is obvious that the six unknown coefficients of the square brackets are really only one unknown coefficient each, c_1 and c_2, and the process of defining them all separately in the model and deducing separate values for them was redundant. Also it should be realised that if c_1 is determined by experiment then there is a severe limitation on c_2. This may only be varied so that the sum of b_1 and b_3 remains constant whilst β, μ, m and α are the same as for c_1. From the meaning of b_1 and b_3 in the model it is clear that both must have the same sign.

COMPARISON WITH EXPERIMENT

There is considerable experimental evidence for turbulent flow and heat transfer in the plain tube and parallel plate channel. If the predictions of Buleev's model do not agree with this evidence for these simple configurations then we must doubt the value of any supposed agreement between theory and experiment for complex situations where the experimental evidence is less well established.

(a) *Plain tube*

The computer programming for applying Buleev's model is straightforward but lengthy and expensive in computer time. Calculations were made of $\varepsilon_{m,r}$ given by

$$\frac{\varepsilon_{m,r}}{\nu} = c\mu\alpha r_o^+ L_o'^2 \int_{-1/\alpha}^{1/\alpha} \frac{L'}{L_o'}\left[\frac{\partial u^+}{\partial r'}\right] \times f_0(q_1\xi) f_1(q_1\xi) G(\xi)\,d\xi \qquad (14)$$

where non-dimensional terms are introduced and the integration is, following Buleev, confined to $-(1/\alpha) < \xi < (1/\alpha)$. This involved an iterative procedure in which an initial estimate is made of the velocity profile (the log law is adequate), $\varepsilon_{m,r}$ is calculated and the velocity gradient recalculated. When a stable result is obtained after six or ten iterations, appropriate integration gives the velocity profile.

By making μ a function of distance from the wall in the sublayer, thickness y_l^+, and by making both μ and y_l^+ functions of Reynolds number it is possible to achieve good agreement with experiment for $\varepsilon_{m,r}$ and for the $u^+ - y^+$ velocity profile. Results are shown in Figs. 1 and 2 and the variation of μ with Reynolds number is shown in Fig. 3. Here the value of μ is that for the main stream where it is constant. The other

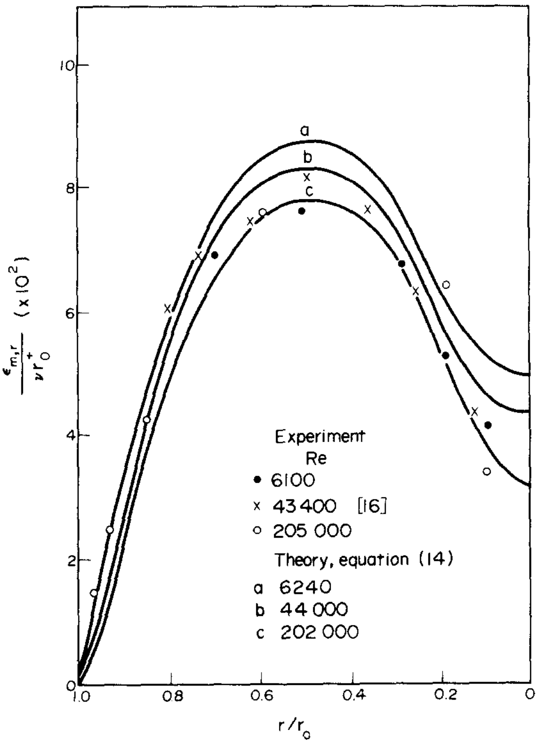

FIG. 1. Buleev's theory prediction and experiment for $\varepsilon_{m,r}$ in a plain tube.

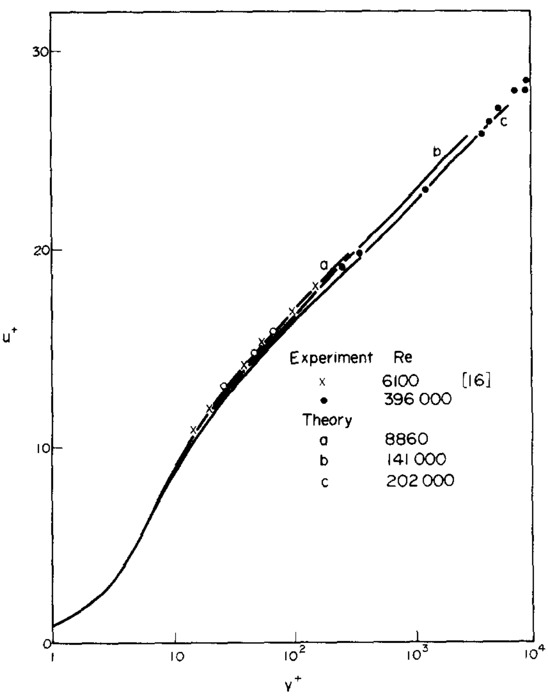

FIG. 2. Buleev's theory and experiment for turbulent velocity profile in a plain tube.

coefficients were given the values suggested by Buleev, since as noted above their values are unimportant provided one coefficient in each group can be varied.

On calculating $\varepsilon_{h,r}$, the appropriate expression is identical to equation (14) except that f_2 replaces f_1. To achieve agreement between theory and experiment

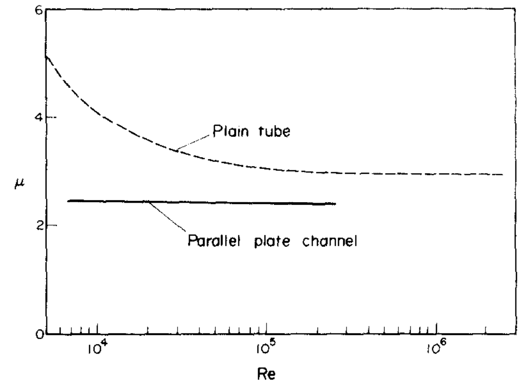

FIG. 3. Variation of coefficient μ with Reynolds number for plain tube and parallel plate channel.

therefore we can only vary the index n and b_1 and b_3 subject to the condition that their sum is the same as that established by the calculation of $\varepsilon_{m,r}$.

Results for $0.01 < Pr < 1000$ for $Re = 8900$ and 200000 are shown in Fig. 4 with $b = 3.5$, $b_3 = 0.9$ and $n = -0.5$ where they are compared with the experimental results of Quarmby and Quirk [10, 11]. The

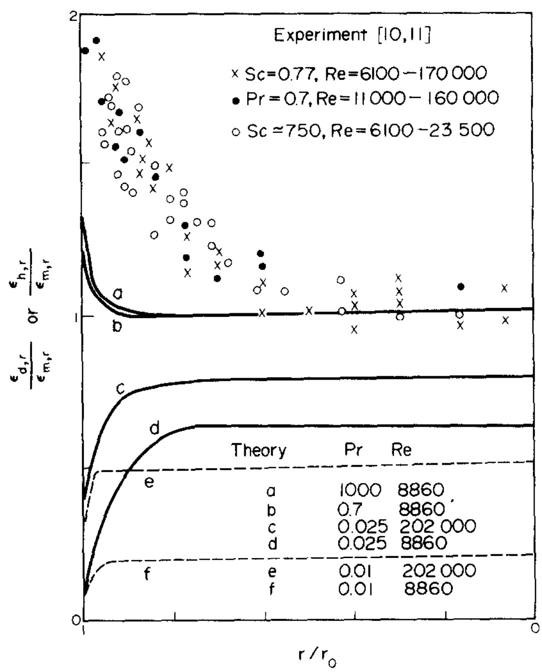

FIG. 4. Comparison of Buleev's theory with experiment for ratio of radial eddy diffusivities in a plain tube.

calculations can be regarded as showing the effect of Pr or of showing the effect of varying the coefficient c_2, equation (13), for a fixed Prandtl number. It was not found possible to improve the agreement between theory and experiment or to achieve the result given by Ramm and Johannsen [6].

Once the coefficients for $\varepsilon_{m,r}$ and $\varepsilon_{h,r}$ are determined the result for $\varepsilon_{h,w}$ the eddy diffusivity in the tangential direction follows without any further possibility of adjustment. The result is shown in Fig. 5. Again the agreement with experiment is unsatisfactory and the theoretical result of Ramm and Johannsen is not produced.

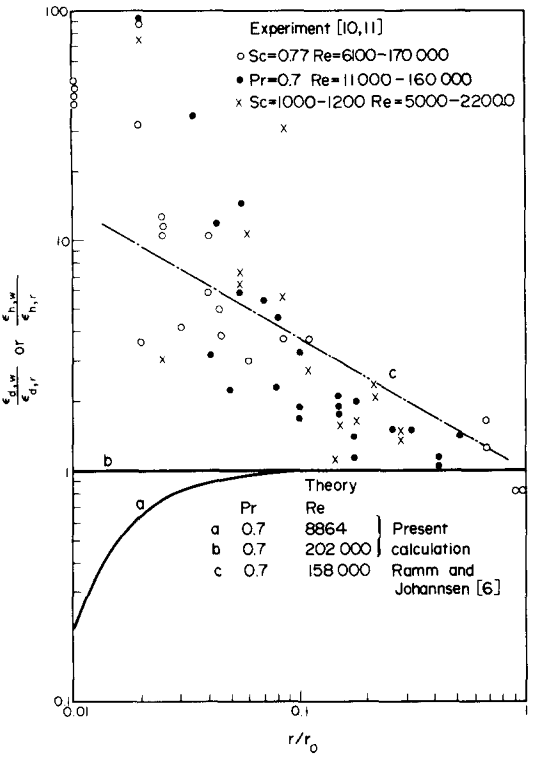

FIG. 5. Comparison of Buleev's theory with experiment for ratio of radial and tangential eddy diffusivities in a plain tube.

Another test of Buleev's model is possible without considering heat or mass transfer. As shown above, a result for $\varepsilon_{m,r}$ can be obtained which agrees well with experiment and this may be taken as establishing the correct values of the unknown coefficients which appear in the Reynolds stress calculations. These values appear also in his expression for $\overline{u'^2}$ and it is a small matter to compute this in the programme for $\varepsilon_{m,r}$. A comparison with the measurement of Laufer [12] showed that the theoretical prediction was out by a factor of up to about forty.

(b) *Parallel plate channel*

For a fully developed turbulent flow in a parallel plate channel Buleev's expressions are very similar to those for the tube except that, y, the cross stream co-ordinate, replaces, r, the radius and the z direction is perpendicular to the plane of the flow. The eddy diffusivity of heat in that direction, $\varepsilon_{h,z}$, has a simple form since velocity, slope and length scale are constant.

Again by making μ a function of distance from the wall but not in this case a function of Reynolds number, it is possible to achieve reasonable agreement between theory and experiment for $\varepsilon_{m,y}$ and for the $u^+ \sim y^+$ profiles. The other coefficients had the values given above but this is insignificant since changing one is equivalent to determining a completely different set. The variation of μ for the parallel plate channel is shown in Fig. 3 and compared with the value for the plain tube. It was intended by Buleev and reiterated by Ramm and Johannsen that the set of coefficients should be the same for all configurations. This is logical since they describe the behaviour of the mole, that is,

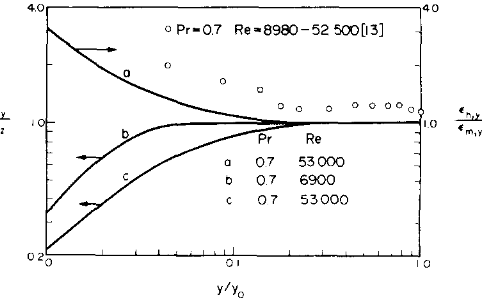

FIG. 6. Comparison of Buleev's theory and experiment for ratio of radial and tangential eddy diffusivities in a parallel plate channel.

the basic mechanism of turbulent exchange. Having to use a different set for the channel is thus a fundamental weakness in the theory.

Figure 6 shows the results for $\varepsilon_{h,y}$ and compares them with the experiments of Page *et al.* [13]. The agreement is the best that could be obtained by adjusting the coefficients subject to the restraints mentioned. The result for $\varepsilon_{h,z}$ also is shown in Fig. 6. Unfortunately, there are no experimental results against which this might be tested but the trend as the wall is approached is, like the corresponding result for the plain tube, contrary to the experimental results for the plain tube. It is unlikely that the effect of the wall in a parallel channel should be different from that in a plain tube; so the theoretical prediction is again opposite to experiment.

(c) *Concentric annuli*

It is not difficult to extend the computer program for the plain tube to calculate $\varepsilon_{m,r}$ and the velocity profile in a concentric annulus. To do this in the simplest way possible and avoid introducing sources of error whose effects could not be identified, initially the programme for the plain tube was extended to make the calculation along the whole of a diameter, so that to calculate the concentric annulus case it was only necessary to make a number of simple changes to the algebra. For example, the shear stress is a function of radius ratio and is different on the inner and outer walls. In this connection, empirical results were used for the radius of zero shear. This is not strictly necessary since given a correct model the radius of zero shear should be predictable along with the other matters of interest. However confidence in the model was not sufficient to justify this expense of computing time. By putting in the radius of zero shear as a function of radius ratio and Reynolds number we avoid considerable computing whilst still being able to evaluate the eddy diffusivities and velocity profiles. A result is shown in Fig. 7 for the eddy diffusivity of momentum for a radius of 2.88. The predictions are seven to twenty times too great and velocity profiles calculated from them have no agreement with experiment. There was no justification for extending the calculation to the eddy diffusivity of heat.

(d) *Rectangular channel*

The calculation of turbulent flow in a rectangular channel by Buleev's theory is rather more complicated

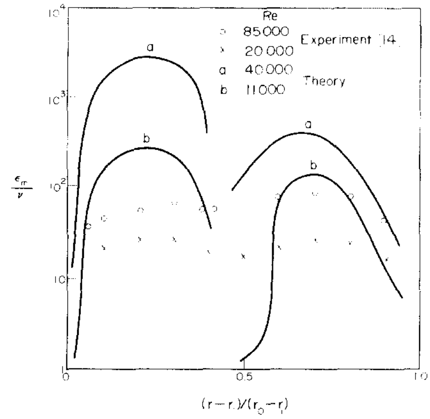

FIG. 7. Prediction and experiment for $\varepsilon_{m,r}$ in a concentric annulus radius ratio 2.88.

than the cases described above. The mean velocity gradient, for example, is given by

$$\frac{\partial V}{\partial n} = \left(\left[\frac{\partial u}{\partial y}\right]^2 + \left[\frac{\partial u}{\partial z}\right]^2\right)^{\frac{1}{2}} \tag{15}$$

and two shear stresses are involved, τ_{xy} and τ_{xz}, so that the link between eddy diffusivities and velocity gradient is a Poisson-type equation. The calculation is rather lengthy. Firstly an estimate is made of $\partial u/\partial y$ and $\partial u/\partial z$ and $\varepsilon_{m,y}$ and $\varepsilon_{m,z}$ calculated from Buleev's expressions. With these values, the equations are solved for the velocity gradients by a finite difference procedure for the whole of the channel. Recalculation of the eddy diffusivities can then be undertaken.

Some results for the velocity profiles in a square duct for $Re = 10000$ and 100000 were obtained which bore some resemblance to measured isovels. The results for the eddy diffusivity were unsatisfactory and in fact the theory predicted that the friction factor increases as Reynolds number decreases. This is clearly wrong as the comparison with the experiments of Hartnett *et al.* [15] shows, Fig. 8.

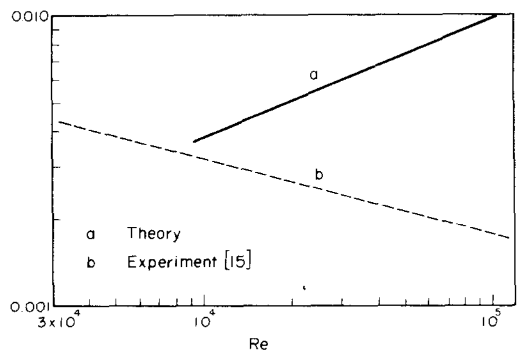

FIG. 8. Predicted friction factors and experiment for a square duct.

DISCUSSION

In all of the calculations care was taken to eliminate errors due to the numerical methods used. Some of the calculations were repeated with twice as many steps. Thus in the tube case two hundred points were taken along the radius on occasion. Different quadrature methods were used to evaluate integrals and the effect of decreasing the step length, that is, increasing the number of quadrature points was investigated. It is believed that the results are accurate calculations of the theory. This is in contrast to Buleev's calculations which themselves involved certain further approximations and simplifications.

It is clear that Buleev's claim to have provided a means of calculating the Reynolds stresses and components of the heat flux vector in a wide variety of cases is not substantiated. The most the theory was able to do was to predict $\varepsilon_{m,r}$ and the velocity profile in two simple cases namely fully developed flow in the plain tube and parallel plate channel. Accordingly it is not more "universal" than Prandtl's mixing length theory. Indeed, examination of the model shows that this is what it is in effect. Thus, although the model starts with a very general picture of turbulent exchange, the simplifications and assumptions made reduce it to a suggestion that the eddy diffusivity in any direction can be calculated from a weighted integral of $l[\partial V/\partial n]$ in that direction. Using such an integral does allow us to calculate the eddy diffusivity at points where the velocity gradient is zero and thus one of the greatest shortcomings of the mixing length theory is avoided. Similarly, using a length scale which is deduced by integrating over the whole area of the flow allows a value to be obtained for ducts of complex shape.

The essential equivalence of Buleev's results and the Prandtl mixing length theory becomes more obvious if we consider the values which the weighting functions take over the range of integration used. The range of integration is $-1/\alpha < \xi < 1/\alpha$, that is, $-L_o < y < L_o$ for any point y_o in the flow. Close to the wall L_o is very close to y_o whatever the shape of the duct. So that integration is taking place over a small distance about y_o. In the centre although the range is greater the variation of parameters is less. In addition, the weighting functions all contain decaying exponentials which severely attenuate contributions to the integral except near the point y_o. All these factors make the integral expression for eddy diffusivity closer and closer to an obvious application of the mixing length theory. This is clear from examination of equations (8) and (9). Table 1 compares the results for $\varepsilon_{m,r}$ for a plain tube

Table 1. Comparison of calculated results for $\varepsilon_{m,r}$ for (a) length scale and velocity gradient functions of ε and (b) length scale and velocity gradient constant

	$L, \frac{\partial V}{\partial n}$ both $f(\xi)$		$L, \frac{\partial V}{\partial n}$ constant	
	Re			
ξ_0	9000	200000	9000	200000
0.97	0.0024	0.0022	0.0022	0.0022
0.93	0.0130	0.0107	0.0128	0.0110
0.90	0.0249	0.0345	0.0242	0.0358
0.85	0.0389	0.0483	0.0392	0.0496
0.75	0.0575	0.0687	0.0587	0.0698
0.5	0.0780	0.0887	0.0797	0.0898
0.25	0.0628	0.0692	0.0643	0.0718

when the calculation is made with L and $\partial V/\partial n$ varying with ξ with those when they are constant and equal to L_o and $[\partial V/\partial n]_o$ respectively. Clearly this latter case is exactly a weighted mixing length calculation and the almost identical results confirm the view that that is what Buleev's theory is in essence.

It is instructive to examine the values which q_2, equation (13), can take since this determines the eddy diffusivity of heat. For any point in the flow, q_2 will vary only with Prandtl number and the effect on $\varepsilon_{h,r}$ was shown in Fig. 4. As Pr is varied this could be regarded as altering the coefficients but keeping Pr fixed so that Fig. 4 shows the effect of changing q_2 for $Pr = 0.7$, say. We cannot alter n arbitrarily since that might give an effect of Pr which was contrary to experiment. It is well established that the ratio $\varepsilon_{h,r}$ to $\varepsilon_{m,r}$ decreases as Pr decreases. Values for liquid metals are less than for air, for example. If the index n could have values greater than one, anomalous results would follow. Table 2 shows the effect of varying the coefficients and Pr and clearly this represents a wide variation of q_2 for $Pr = 0.7$. The result for $Pr = 1000$ is about the limit of what can be done to achieve agreement between Buleev's theory and experiment. This is

Table 2. Effect of altering coefficients b_1, b_3 and n on ratio $\varepsilon_{h,r}/\varepsilon_{m,r}$ in a plain tube

$\varepsilon_{h,r}/\varepsilon_{m,r}$ $\xi = 0.99$	$\xi = 0.9$	b_1	b_3	n
1·165	1·056	0·93	3.51	-0.5
1.205	1.068	0.24	4.2	-0.5
1.199	1.066	0.34	4.1	-0.5
1.193	1.064	0.44	4.0	-0.5
1.148	1.050	1.24	3.2	-0.5
1.132	1.045	1.54	2.9	-0.5
1.165	1.055	0.93	3.51	-0.75
1.165	1.055	0.93	3.51	0.33
1.165	1.055	0.93	3.51	0.2

easily seen since the theoretical line approaches the experiment as Pr increases and q_2 decreases. Since q_2 is the argument of an exponential function, increasing Pr makes the function unity which is, of course, independent of the values of the coefficients. Another point of significance in the assessment of Buleev's theory is that, for $Pr = 1.0$, $\varepsilon_{h,r}$ is equal to $\varepsilon_{m,r}$ since q_1 and q_2 become identical as do equations (8) and (10). This result is independent of the values of the coefficients. Thus Buleev's theory reduces to Reynolds' analogy for $Pr = 1.0$ and gives the ratio of $\varepsilon_{h,r}$ to $\varepsilon_{m,r}$ as unity. Clearly this is a very disturbing anomaly since it would still have occurred had it been possible to achieve good agreement with experiment for $Pr = 0.7$; that is, to predict the ratio as greater than unity.

Further on examining the effect of q_2 in the calculation of $\varepsilon_{h,w}$, it is clear that the result claimed by Ramm and Johannsen [6], that the ratio $\varepsilon_{h,w}$ to $\varepsilon_{h,r}$ is greater than unity and in agreement with experiment, Fig. 5, is impossible. Any point on the radius, ξ_o, the value of the integral for positive values of ξ, that is towards the wall, is less than for negative values, that is towards the centre. So that the contribution of the integral for $0 < \xi < 1/\alpha$ to equation (10) can be negligible compared with the contribution of the integral for $-1/\alpha < \xi < 0$. This is made clear by considering the argument q_2 which can be rewritten in non-dimensional terms as

$$q_2 = \frac{12m}{\mu\alpha}\frac{(b_1 Pr^{1-n} + b_3)}{r_o^+ L_o^2}\frac{1}{\left(\dfrac{L}{L_o}\right)^3\left(\dfrac{\partial u}{\partial \xi}\right)}. \tag{16}$$

Significant variations in q_2 are due to variations in L. Thus, at a point near the wall, for ξ positive, $L < L_o$ and accordingly q_2 is large and the negative exponential weighting function severely attenuates the integrand.

Table 3. Values of the product of the weighting functions f_0, f_1 and ϕ at the Gaussian quadrature points for different wall distances

n	$y^+ = 3$	12	42	135
4	0	0	3.90–8	3.57–3
3	0	0	4.38–5	1.03–2
2	0	1.47–8	6.55–3	4.45–2
1	0	1.49–2	1.28–1	2.13–1
-1	1.52–3	1.31–1	2.22–1	2.34–1
-2	5.32–4	3.67–2	6.56–2	6.70–2
-3	3.40–4	1.56–2	2.73–2	2.65–2
-4	2.65–4	9.70–3	1.74–2	1.47–2

For ξ negative, $L_o < L$ and the reverse is true. Table 3 gives the values of the product of $f_0(\xi)$, $f_1(\xi)$ and $G(\xi)$ at the n quadrature points of a nine point Gaussian formula. The values are given in floating point form with the figure on the right of each column giving the exponent. A zero indicates a value less than 10^{-10}.

When a calculation is made of $\varepsilon_{h,w}$, equation (11), for the case of the plain tube, the integration is along a chord, that is, towards the wall and L, at each point ζ, will be less than L_o. This is true for both halves of the integration. Accordingly, the integral $-1/\alpha < \eta < 1/\alpha$ producing $\varepsilon_{h,w}$ will always be less than the integral $-1/\alpha < \xi < 1/\alpha$ producing $\varepsilon_{h,r}$ and the ratio of $\varepsilon_{h,w}$ to $\varepsilon_{h,r}$ will always be less than unity except at the centre of the tube where it will be identically unity.

Finally, it should be realised that, as equation (6) shows, the Buleev model is one in which the turbulence is functionally dependent on the mean field only. It is known, Bradshaw [15], that approximations in which the turbulence is closed on itself are in far better accord with the physics of the motion. Accordingly, it would not seem to be worthwhile to attempt to improve the Buleev model either by modifications to the exchange process or, like Ramm and Johannsen [4, 7], by using special definitions of the length scales chosen to fit each particular case.

CONCLUSIONS

A careful assessment of Buleev's analysis of turbulent exchange shows several steps which are illogical or inconsistent so that the justification for the model can only be heuristic. Further, although the model is apparently quite general the assessment shows that the simplifications introduced in applying it to specific cases reduce it to a slightly complicated variation of Prandtl's mixing length model. On applying the model to the prediction of turbulent flow and heat transfer in four simple cases it seems that the model works where Prandtl's model works but not otherwise. Thus it is possible to adjust the coefficients of the model to achieve good agreement between prediction and experiment for $\varepsilon_{m,r}$ and the velocity profile in a plain tube but poor agreement follows when these coefficients are used to predict other turbulence quantities in a plain tube. Different coefficients allow good agreement to be achieved for a parallel plate channel for the corresponding quantities, $\varepsilon_{m,y}$ and $u^+ \sim y^+$. However on using the first set of coefficients for a plain concentric annulus and the second for a square duct completely unsatisfactory results are obtained and it is clear that the coefficients would have to be adjusted further to give good agreement with experiment. Accordingly Buleev's claim to have produced a "universal" model of turbulent momentum exchange applicable to ducts of arbitrary cross-section is not substantiated.

It is not substantiated for heat or mass exchange either since there is poor agreement between prediction and experiment for the radial and tangential eddy diffusivities of heat or mass in a plain tube. This agreement cannot be improved by adjusting the one further coefficient allowed by the model and indeed, examination of the form of the equations resulting from the model shows that it cannot give the correct variation near the wall of the ratio of these quantities.

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APPENDIX

Buleev's expressions for the terms of the turbulent stress tensor are:

$$-\rho\overline{v_i'v_i'} = -\rho P_{ii} + 2\rho\varepsilon_m^{ii}\frac{\partial\bar{v}_i}{\partial x_i} - \sum_j \rho\omega^{jj}\left(\frac{\partial\bar{v}_i}{\partial x_j}\right)^2 \tag{1}$$

$$-\rho\overline{v_i'v_k'} = -\rho\varepsilon_m^{ii}\frac{\partial\bar{v}_k}{\partial x_i} + \rho\varepsilon_m^{kk}\frac{\partial\bar{v}_i}{\partial x_k} - \sum \rho\omega_m^{jj}\frac{\partial\bar{v}_i}{\partial x_j}\frac{\partial\bar{v}_k}{\partial x_k} \tag{2}$$

where

$$P_{ik} = \mu^2\int L^2\left[\frac{\partial V}{\partial n}\right]f_0^2\phi\cos(s,x_l)\cos(s,x_k)\,\mathrm{d}\tau \tag{3}$$

$$\varepsilon_m^{lj} = \mu\int_D L\left[\frac{\partial V}{\partial n}\right]f_0 f_1 s\phi\cos(s,x_l)\cos(s,x_j)\,\mathrm{d}\tau \tag{4}$$

$$\omega_m^{lj} = \int_D f_1^2 s^2\phi\cos(s,x_l)\cos(s,x_j)\,\mathrm{d}\tau \tag{5}$$

whilst L is a mixing length, $\partial V/\partial n$ is the mean velocity gradient and f_0, f_1 and ϕ are functions of the turbulence model. For the terms of the heat flux vector he gives

$$-c_p\rho\overline{v_i'T'} = c_p\rho\varepsilon_n^{ii}\frac{\partial\bar{T}}{\partial x_i} - \sum_j c_p\rho\omega_n^{jj}\frac{\partial\bar{v}_i}{\partial x_j}\frac{\partial\bar{T}}{\partial x_j} \tag{6}$$

where

$$\varepsilon_h^{lj} = \mu\int_D L\left[\frac{\partial V}{\partial n}\right]f_0 f_2 s\phi\cos(s,x_l)\cos(s,x_j)\,\mathrm{d}\tau \tag{7}$$

and

$$\omega_n^{lj} = \int_D f_1 f_2 s^2\phi\cos(s,x_l)\cos(s,x_j)\,\mathrm{d}\tau. \tag{8}$$

In the last equation f_2 also arises from the turbulence model.

The turbulence model postulates that at any point M_o the instantaneous fluctuations are related to those at any other point, M, by

$$u'(M_o) = u'(M)f_0 + [\bar{u}(M) - \bar{u}(M_o)]f_1 \tag{9a}$$

$$w'(M_o) = w'(M)f_0 + [\bar{u}(M) - \bar{u}(M_o)]f_1 \tag{9b}$$

and

$$T'(M_o) = [\bar{T}(M) - \bar{T}(M_o)]f_2 \tag{9c}$$

so that

$$\begin{aligned}\rho u'(M_o)w'(M_o) &= \{u'(M)f_0 + [\bar{u}(M) - \bar{u}(M_o)]f_1\}\\ &\quad\times\{w'(M)f_0 + [\bar{w}(M) - \bar{w}(M_o)]f_1\}\\ &= F_1(MM_o)\end{aligned} \tag{10a}$$

and

$$\rho c_p u'(M_o)T'(M_o) = c_p\{u'(M)f_0 + [\bar{u}(M) - \bar{u}(M_o)]f_1\} \times \{[\bar{T}(M) - \bar{T}(M_o)]f_2\} = F_2(MM_o). \quad (10b)$$

During a flight from M to M_o the instantaneous value u^*, w^* and T^* change according to

$$du^* = \frac{3}{R}A_1(\bar{u} - u^*)dt \quad (11a)$$

$$dw^* = \frac{3}{R}A_1(\bar{w} - w^*)dt \quad (11b)$$

$$dT^* = \frac{3}{R}A_2(\bar{T} - T^*)dt \quad (11c)$$

whilst the diameter, mean free path and instantaneous velocity fluctuations are given by:

$$d = \beta L \quad (12a)$$

$$\lambda = \alpha L \quad (12b)$$

$$V' = \mu L\left[\frac{\partial V}{\partial n}\right]. \quad (12c)$$

Solution of equations (9–11) of this section identifies f_0, f_1 and f_2 as

$$f_0 = \exp(-p_1 s) \quad (13a)$$

$$f_1 = [1 - \exp(-p_1 s)]/p_1 s \quad (13b)$$

$$f_2 = [1 - \exp(-p_2 s)]/p_2 s \quad (13c)$$

where p_1 is $3A_1/RV'$, p_2 is $3A_2/RV'$ and s is the distance from M to M_o.

The weighting function, ϕ, derived from the analogy with neutron flux attenuation, is given by:

$$\phi = \frac{c}{4\pi s^2}\exp(-k_0 s). \quad (14)$$

The length scale or mixing length is defined by an average over the area of flow so that L in equations (12) above is given by:

$$\frac{1}{L} = \int_0^{2\pi}\frac{1}{l}d\phi \quad (15)$$

where l is the distance from the point in question to the tube wall along the direction of angle ϕ.

SUR LE MODELE DE BULEEV DU TRANSFERT TURBULENT

Résumé—Un modèle "universel" du transfert turbulent dû à Buleev [1] est examiné, modèle qui fournit des expressions pour toutes les composantes du tenseur des contraintes turbulentes et du vecteur flux de chaleur ou de masse. Des applications récentes de ce modèle dans des cas variés ayant donné des résultats satisfaisants, une plus grande attention lui a été accordée. On compare ici avec l'expérience les prévisions des diffusivités turbulentes de quantité de mouvement, de chaleur ou de masse dans des écoulements présentant plusieurs configurations différentes. Les calculs et comparaisons ne confirment pas l'exigence d'universalité. En particulier, les diffusivités turbulentes tangentielles de chaleur ou de masse calculées dans un tube lisse sont très différentes des valeurs expérimentales et une limitation inhérente au modèle lui-même interdit toute amélioration.

EINE AUSWERTUNG DES BULEJEW—MODELLS FÜR TURBULENTEN AUSTAUSCH

Zusammenfassung—Für ein "universelles" Modell des turbulenten Austausches nach Bulejew wird eine Auswertung durchgeführt; es werden dabei alle Komponenten des turbulenten Spannungstensors und des Wärme- und Stoffstromvektors herangezogen. Kürzliche erfolgreiche Anwendungen dieses Modells lassen es beachtenswert erscheinen. Die berechneten Werte für den turbulenten Impulsaustausch und für den Wärme- oder Massenstrom in verschiedenen Anordnungen werden mit Experimenten verglichen. Die Auswertung und der Vergleich unterstützen nicht den Anspruch auf Universalität. Besonders stark sind die Unterschiede zwischen den experimentellen Ergebnissen und den berechneten Werten für tangentiale turbulente Ausbreitung von Wärme oder Masse in einem einfachen Rohr. Eine dem Modell anhaftende Beschränkung verhindert Verbesserungen.

ОЦЕНКА МОДЕЛИ ТУРБУЛЕНТНОСТИ БУЛЕЕВА

Аннотация — Проведена оценка «универсальной» модели турбулентного переноса [1], которая позволяет получить выражения для всех компонентов тензора турбулентных напряжений и вектора потока тепла или массы. Последние успешные использования данной модели в различных случаях привлекли к ней внимание. В данной статье расчетные коэффициенты турбулентного переноса количества движения и тепла или массы в различных моделях течения сравниваются с экспериментальными. Оценка и сравнение не подтверждают полной универсальности модели. В частности, расчетные тангенцианальные коэффициенты переноса тепла или массы в трубе простой конфигурации существенно отличаются от экспериментальных. Присущие модели ограничения не позволяют усовершенствовать её.